Experimental Design 44.3

Introduction

In Sections 44.1 and 44.2 we have considered how to analyse data from experiments of certain types. Of course, before we can analyse any data we must conduct the experiment and before we can conduct an experiment we must design it. The work of applying statistical ideas to engineering experiments does not begin with the analysis of data. It begins with the design of the experiment. It is important to give proper consideration to experimental design to make sure that our experiment is efficient and that it will, in fact, give us the information we require. A badly designed experiment may give poor or misleading results or may turn out to be an expensive waste of time and money.

Prerequisites

Before starting this Section you should ...

- explain the concepts and terminology of the one-way and two-way ANOVA
- be familiar with the F-distribution
- understand the general techniques of hypothesis testing

Learning Outcomes

On completion you should be able to ...

- explain the basic concepts of experimental design
- apply randomized blocks and Latin Square designs
- analyse the results from randomized blocks and Latin Square designs

1. Experimental design

So far in this Workbook we have looked at some of the statistical methods used in the analysis and interpretation of experimental results. There are occasions when the planning of an experiment is not in the control of the statistician responsible for analysing the results. It is always preferable to have some idea of the likely variability of the data so that any experimental design can take this into account. For this reason, the **design of experiments** is of crucial importance if weight is to be given to the results obtained. Usually, the experimenter will have to take into account:

(a) The definition of the problem to be investigated. This would usually include the selection of the response variable to be measured and the factors or treatments influencing the response. Remember that the factors may be quantitative (such as temperature, pressure or force), qualitative (such as days of the week, machine operators or machines themselves) and decisions must be taken as to whether these factors are *fixed* or *random* and at what levels they are to be used.

(b) The sample size. Clearly the experimenter should determine the number of observations to be taken and the random order in which the experiments are to occur in order that the effects of uncontrollable or unforeseen variables are minimized.

(c) Data collection. Decisions need to be taken as to how the data are to be collected and tabulated. The calculation of the test statistics needs to be taken into account as does the level of acceptance or rejection of any hypotheses used.

We have already looked at some introductory ANOVA situations and we now turn our attention to so-called **block designs** used in the conduct of experiments.

Block design

Block design, or more specifically, **randomized block design** enables an unbiased estimate of error to be obtained and ensures that the error obtained is a minimum.

As an illustration, imagine that we wish to compare four extrusion processes and measure their effect on the brittleness of copper wire produced. Assume further that the copper from which the wire is made is delivered in quantities to allow only four tests per batch. We will refer to such a batch as a **block**. If we replicate each treatment four times, we could organize the four members of block 1 all to receive treatment A, all four members of block 2 all to receive treatment B and so on. This situation could be represented as shown below.

Block	**Treatment**			
1	A	A	A	A
2	B	B	B	B
3	C	C	C	C
4	D	D	D	D

Giving every member of a given block the same treatment is not a very sensible thing to do. The reason for this is that any observed differences might well be due to differences between the blocks and not differences between the treatments. Remember that each block consists of a batch of material and so as engineers we would expect some variation in batches of materials delivered for processing. We can avoid this pitfall by ensuring that the blocks are distributed among the treatments. Ensuring that each block experiences every treatment could lead to the situation represented below.

Block	Treatment			
1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D

On the face of it, we appear to have a sensible way of organizing the treatments between blocks. However it is entirely possible that other variables might come into play which might change with time which would bias the observations made. For example, if the treatments are always applied in a particular order, say A, B, C and then D it could be that the state of the extrusion machines might change with time which would bias the results. For example, if treatment A is always applied first, it could be that the extrusion machine is not fully 'warmed up' and so non-typical results might occur.

In order to remove the bias referred to above, we could randomize the order in which the treatments are applied. This might result in the **randomized block design** represented below in which comparisons are made between sets of treatments applied to fairly homogeneous material.

Block	Treatment			
1	A	B	D	C
2	B	D	A	C
3	C	B	D	A
4	A	D	B	C

Note that this design may be taken to consist of two independent variables, namely blocks and treatments. The total sum of the squares (previously referred to as SS_T) may be partitioned as

$$SS_T = SS_{\text{Blocks}} + SS_{\text{Treatments}} + SS_{\text{Error}}$$

We illustrate this in Example 2.

Example 2

The compressive strength of concrete to be used in the construction of a dam may depend on the proportion of a particular component of the mix. In order to investigate this, the compressive strength of concrete containing a variety of percentages of the component, 2%, 4%,.....10% was measured. The concrete was made using four batches of cement. One part of each batch was used with each of the percentages of the component of interest. The resulting concrete samples were then subjected to tests to determine their compressive strength. The data obtained are given below.

	Compressive Strength			
	Block (Batch)			
Component %	**1**	**2**	**3**	**4**
2	201	185	182	179
4	200	195	220	199
6	257	240	224	225
8	252	228	275	250
10	280	275	277	260

What conclusions may be drawn about the effect of the mix component on the compressive strength of the resulting concrete? Is there evidence of differences between the batches of cement. Use the 5% level of significance.

Solution

Assuming that no interaction takes place between the blocks and the parts, we can calculate the appropriate sums of squares as done previously. The grand total of the observations is represented by T.

	Compressive Strength				
Block					
Component %	**Part**				**Totals**
	1	**2**	**3**	**4**	
2	201	185	182	179	747
4	200	195	220	199	814
6	257	240	224	225	946
8	252	228	275	250	1005
10	280	275	277	260	1092
Totals	1190	1123	1178	1113	$T = 4604$

The total sum of squares is given by

$$SS_T = 201^2 + \ldots + 182^2 + \ldots + 260^2 - \frac{4604^2}{5 \times 4} = 1082234 - 1059840.8 = 22393.2$$

The sum of squares for the treatments is given by

$$SS_{Tr} = \frac{1190^2 + \ldots + 1113^2}{5} - \frac{4604^2}{5 \times 4} = 1060736.4 - 1059840.8 = 895.6$$

Solution (contd.)

The sum of squares for the blocks is given by

$$SS_{Bl} = \frac{747^2 + \ldots + 1092^2}{4} - \frac{4604^2}{5 \times 4} = 107950.5 - 1059840.8 = 19661.7$$

The sum of squares for the errors is given by

$$SS_E = SS_T - SS_{Tr} - SS_{Bl} = 22393.2 - 895.6 - 19661.7 = 1835.9$$

These calculations give rise to the following ANOVA table

Source of Variation	**Sum of Squares**	**Degrees of Freedom**	**Mean Squares**	**Value of F**
Blocks	895.6	3	298.53	$F_{Tr} = \dfrac{SS_{Tr}}{SS_E} = 1.95$
Treatments	19661.7	4	4915.43	$F_{Bl} = \dfrac{SS_{Bl}}{SS_E} = 32.13$
Error	1835.9	12	152.99	
Total	22393.2	19		

Conclusions

(a) **Blocks**

From the F-tables, $F_{3,12} = 3.49$ and since $1.95 < 3.49$ we have no evidence to reject the null hypothesis that there are no differences between the batches of cement.

(b) **Treatments**

From the F-tables, $F_{4,12} = 3.26$ and since $32.13 > 3.26$ we have sufficient evidence to reject the hull hypothesis that the addition of the mix component has no effect on the compressive strength of the resulting concrete. Hence our conclusion (at the 5% level of significance) is that the addition of the mix component does affect the compressive strength of the resulting concrete.

Task

The tensile strength of aluminium alloy tubing to be used in the construction of aircraft is known to depend on the extrusion process by which the tubing is produced. In order to investigate the tensile strength of the alloy made by four different extrusion processes, four large samples were made using each extrusion process. One sample from each process was sent to each of four laboratories for measurement of its tensile strength. The data obtained are given below.

	Block (lab)			
Process	**1**	**2**	**3**	**4**
1	202	186	182	180
2	200	195	224	199
3	259	243	225	223
4	252	227	276	251

What conclusions may be drawn about the effect of the extrusion processes on the tensile strength of the resulting tubing? Is there sufficient evidence to suggest that there are systematic differences between measurements from the different laboratories? Use the 5% level of significance.

Your solution

Answers

Assuming that no interaction takes place between the factors, we can calculate the appropriate sums of squares, representing the grand total by T.

	Blocks (lab)				
Process	**1**	**2**	**3**	**4**	**Totals**
1	202	186	182	180	750
2	200	195	224	199	818
3	259	243	225	223	950
4	252	227	276	251	1006
Totals	913	851	907	853	$T = 3524$

The total sum of squares is given by

$$SS_T = 202^2 + \ldots + 182^2 + \ldots + 251^2 - \frac{3524^2}{4 \times 4} = 789420 - 776161 = 13259$$

The sum of squares for the treatments is given by

$$SS_{Tr} = \frac{913^2 + \ldots + 853^2}{5} - \frac{3524^2}{4 \times 4} = 3102808 - 776161 = 846$$

The sum of the squares for the blocks is given by

$$SS_{Bl} = \frac{750^2 + \ldots + 1006^2}{4} - \frac{3524^2}{4 \times 4} = 786540 - 776161 = 10379$$

The sum of squares for the errors is given by

$$SS_E = SS_T - SS_{Tr} - SS_{Bl} = 13259 - 846 - 10379 = 2034$$

These calculations give rise to the following ANOVA table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	Value of F
Blocks (labs)	846	3	282	$F_{Tr} = \dfrac{SS_{Tr}}{SS_E} = 1.25$
Treatments	10379	3	3459.67	$F_{Bl} = \dfrac{SS_{Bl}}{SS_E} = 15.31$
Error	2034	9	226	
Total	13259	15		

Conclusions

(a) **Blocks**

From F-tables, $F_{3,9} = 3.86$ and since $1.25 < 3.86$ we have no evidence to reject the null hypothesis that there are no systematic differences between measurements from the different laboratories.

(b) **Treatments**

From the F-tables, $F_{3,9} = 3.86$ and since $15.31 > 3.86$ we have sufficient evidence to reject the hull hypothesis that the extrusion process has no effect on the tensile strength of the tubing. Hence our conclusion (at the 5% level of significance) is that the extrusion process does affect the tensile strength of the aluminium tubing.

Latin Squares

We have previously looked at the elimination of a source of variation via the use of randomized blocks. If, in a given situation, there are two sources of variation to be controlled, a Latin Square design may provide the best possible analysis. Essentially this design groups the treatments involved into different blocks simultaneously. As an example, consider the problem of checking the quality of the output of four machines over four 6-hour shifts manned by the same four operators on each shift. A Latin Square design allocates the sixteen combinations to be used. The machines are labelled A, B, C and D.

A design is as follows.

		Operator			
		1	**2**	**3**	**4**
	1	A	B	C	D
Shift	**2**	B	C	D	A
	3	C	D	A	B
	4	D	A	B	C

Notice that each machine appears in each row and each column exactly once. Notice also that the number of observations must be equal to the square of the number of treatments (here $4^2 = 16$), or a multiple of this. In situations where a large number of treatments are used, a very substantial testing effort is implied. In return, a high reduction in errors is achieved since every row and every column is a complete replication. Experiments using Latin Squares should be designed so that the differences in rows and columns represent the major sources of variation to be considered.

Example 3

In an experiment designed to compare the tensile strengths of plastic tubes manufactured by different methods there are four different methods, A, B, C, D. It is also believed that there may be effects due to differences between batches of the plastic pellets which are used as raw material and between manufacturing plants. These factors are arranged in a **Latin Square** with four different manufacturing plants as the row factor and the pellet batch as the column factor.
Analyse the results (in coded units of tensile strength), given below, to determine whether or not there are significant differences between the methods. Use the 5% level of significance.

		Pellet batch			
		1	**2**	**3**	**4**
Plant	**1**	A 16.6	B 16.9	C 17.4	D 17.4
	2	D 17.1	C 16.8	B 19.2	A 16.6
	3	C 17.4	D 17.0	A 16.8	B 19.2
	4	B 18.6	A 17.4	D 17.4	C 19.2

Solution

The treatment totals are as follows:

Method	A	B	C	D
Total	67.4	73.9	70.8	68.9

The row totals are as follows:

Plant	1	2	3	4
Total	68.3	69.7	70.4	72.6

The column totals are as follows:

Batch	1	2	3	4
Total	69.7	68.1	70.8	72.4

The grand total is 281.0. The total sum of squares is

$$16.6^2 + \cdots + 19.2^2 - \frac{281.0^2}{16} = 13.2375.$$

The treatments sum of squares is $\dfrac{1}{4}(67.4^2 + \cdots + 68.9^2) - \dfrac{281.0^2}{16} = 5.8925.$

The batches sum of squares is $\dfrac{1}{4}(69.7^2 + \cdots + 72.4^2) - \dfrac{281.0^2}{16} = 2.4625.$

The plants sum of squares is $\dfrac{1}{4}(68.3^2 + \cdots + 72.6^2) - \dfrac{281.0^2}{16} = 2.4125.$

The analysis of variance table is as follows.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio
Treatments	3	5.8925	1.9642	4.77
Batches	3	2.4625	0.8208	1.99
Plants	3	2.4125	0.8042	1.95
Residual	6	2.4700	0.4117	
Total	15	13.2375		

The upper 5% point of the $F_{3,6}$ distribution is 4.76. Thus we can draw the following conclusions.

Treatments (manufacturing methods) The variance ratio is significant at the 5% level so we conclude that there is sufficient evidence to reject the null hypothesis that the treatments give equal mean tensile strengths.

Batches The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the batches have no effect on mean tensile strengths.

Plants The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the plants have no effect on mean tensile strengths.

Task

The yields of a chemical process when using three different catalysts, A, B, C, are to be compared. It is also believed that there may be effects due to the reaction vessel used and the operator. A Latin Square design is used with three operators and three vessels. A batch is produced using each combination of catalyst, vessel and operator. The results (%) are as follows.

		Reaction vessel		
		1	**2**	**3**
Operator	**1**	A 81.4	B 63.9	C 59.6
	2	B 61.3	C 48.6	A 68.5
	3	C 58.3	A 70.2	B 72.5

Analyse the results to test for the effects of the factors. Use the 5% level of significance.

Your solution

Answer

The treatment totals are as follows:

Catalyst	A	B	C
Total	220.1	197.7	166.5

The row totals are as follows:

Operator	1	2	3
Total	204.9	178.4	201.0

The column totals are as follows:

Vessel	1	2	3
Total	201.0	182.7	200.6

The grand total is 584.3. The total sum of squares is

$$81.4^2 + \cdots + 72.5^2 - \frac{584.3^2}{9} = 722.3556.$$

The treatments sum of squares is

$$\frac{1}{3}(220.1^2 + 197.7^2 + 165.5^2) - \frac{584.3^2}{9} = 483.1289.$$

The operators sum of squares is

$$\frac{1}{3}(204.9^2 + 178.4^2 + 201.0^2) - \frac{584.3^2}{9} = 136.4689.$$

The reaction vessels sum of squares is

$$\frac{1}{3}(201.0^2 + 182.7^2 + 200.6^2) - \frac{584.3^2}{9} = 72.8289.$$

The analysis of variance table is as follows.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio
Treatments (catalysts)	2	483.1289	241.56445	16.143
Operators	2	136.4689	68.23445	4.560
Reaction vessels	2	72.8289	36.41445	2.433
Residual	2	29.9289	14.96445	
Total	8	722.3556		

The upper 5% point of the $F_{2,2}$ distribution is 19.00.

Answer continued

Thus we can draw the following conclusions.

(a) **Treatments (catalysts)**

The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the treatments give equal mean yields.

(b) **Operators**

The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the operators have no effect on mean yields.

(c) **Reaction vessels**

The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the reaction vessels have no effect on mean yields.

Note that, with only two degrees of freedom in the residual, this design gives poor power to the tests.

Exercises

1. Aluminium is produced industrially by electrolysis in reduction cells. The 'current efficiency' of a reduction cell is the yield of aluminium as a percentage of the yield predicted in an ideal cell by Faraday's law. Improvements in the computer control system could improve current efficiency. Current efficiency also varies from one cell to another. In an experiment, three control schemes were compared. Each control scheme was applied to each of ten cells in random order. The current efficiency was measured over one week's operation in each case. The data are as follows. Here the cells are 'blocks.'

Control	**Cell (block)**									
Scheme	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	80.27	79.44	81.59	79.78	80.39	81.92	82.87	82.04	83.41	84.52
B	84.31	83.33	86.57	84.49	84.15	85.45	85.05	83.62	85.96	85.62
C	83.59	80.36	84.55	80.03	81.59	80.75	82.61	85.20	84.29	81.60

What conclusions can be drawn about the effect of the control schemes on current efficiency? Use the 5% level of significance.

Exercises continued

2. Plastic tubing is made by an extrusion process, starting with the plastic in pellet form. The tensile strength of the tubes may depend on which of four extrusion processes is used. Tubes were made using each of the four methods and samples tested for tensile strength. Four different batches of plastic pellets were used and each method was used once with plastic from each batch. The data are given below.

Batch	**Tensile Strength Method**			
(Block)	**1**	**2**	**3**	**4**
1	202	186	182	180
2	200	195	224	199
3	259	243	225	223
4	252	227	276	251

What conclusions may be drawn about the effect of the extrusion process on the resulting tubing? Use the 5% level of significance.

3. Crash tests with dummies are used to investigate the effects of different car seat-belt mechanisms. The response variable of interest is the maximum acceleration, in units of g, of the dummy's head. Four different variations of the mechanism, A, B, C, D, are compared in a Latin Square design with four dummies and four different impact angles, but all at the same impact speed.

The data are as follows:

		Dummy			
		1	**2**	**3**	**4**
Impact Angle	**1**	B 3.84	C 3.75	D 4.26	A 3.97
	2	A 3.44	B 3.93	C 4.18	D 3.36
	3	C 3.82	D 3.04	A 3.77	B 3.34
	4	D 3.43	A 2.93	B 3.77	C 3.41

Use an analysis of variance to test for the effects of the three factors. Use the 5% level of significance. Treat all three factors as "fixed."

What assumptions have to be made to justify your analysis?

How might the design be altered to allow investigation of the possibility of interaction effects between the factors?

Answers

1. Subtracting 80 from every observation will have no effect on the analysis of variance. The data now become as follows.

Control Scheme	Cell (block) 1	2	3	4	5	6	7	8	9	10	Total
A	0.27	-0.56	1.59	-0.22	0.39	1.92	2.87	2.04	3.41	4.52	16.23
B	4.31	3.33	6.57	4.49	4.15	5.45	5.05	3.62	5.96	5.62	48.55
C	3.59	0.36	4.55	0.03	1.59	0.75	2.61	5.20	4.29	1.60	24.57
Total	8.17	3.13	12.71	4.30	6.13	8.12	10.53	10.86	13.66	11.74	89.35

The total sum of squares for the 30 observations is given by

$$SS_T = 0.27^2 + \ldots + 1.60^2 - \frac{89.35^2}{30} = 388.5413 - \frac{89.35^2}{30} = 122.400$$

The sum of squares for the three control schemes is given by

$$SS_S = \frac{16.23^2 + 48.55^2 + 24.57^2}{10} - \frac{89.35^2}{30} = \frac{3224.2003}{10} - \frac{89.35^2}{30} = 56.306$$

The sum of squares for the ten cells (blocks) is given by

$SS_B = \dfrac{8.17^2 + \ldots + 11.74^2}{3} - \dfrac{89.35^2}{30} = \dfrac{913.3349}{3} - \dfrac{89.35^2}{30} = 38.331$ The residual sum of squares is given by

$$SS_E = SS_T - SS_S - SS_B = 122.400 - 56.306 - 38.331 = 27.763$$

These calculations lead to the following ANOVA table.

Source of Variation	Deg. of Freedom	Sum of Squares	Mean Square	Variance Ratio
Control Schemes	2	56.306	28.153	18.25
Cells (blocks)	9	38.311	4.259	2.76
Residuals (error)	18	27.763	1.542	
Total	29	122.400		

The following conclusions may be drawn.

Control Schemes

From F-tables $F_{2,18} = 3.55$ and since 18.25>3.55 we have sufficient evidence to reject the null hypothesis that there is no difference in effect between the control schemes. It appears that Control Scheme B gives the greatest current efficiency.

Cells (blocks)

From F-tables $F_{9,18} = 2.46$ and since 2.76>2.46 we do not have sufficient evidence to reject the null hypothesis that there is no difference in mean current efficiency between the cells. Allowing for the cell effect may, nevertheless, have improved the sensitivity of the test for control scheme effects.

Answers continued

2. Since there is only one observation on each combination of batch and extrusion method, we must assume that there is no interaction between the factors. We can calculate the sums of squares as follows.

Batch (Block)	**Tensile Strength Method 1**	**2**	**3**	**4**	**Total**
1	202	186	182	180	750
2	200	195	224	199	818
3	259	243	225	223	950
4	252	227	276	251	1006
Total	913	851	907	853	3524

The total sum of squares is given by

$$SS_T = 202^2 + \ldots + 251^2 - \frac{3524^2}{16} = 789420 - 776161 = 13259$$

The sum of squares for the extrusion methods is given by

$$SS_M = \frac{913^2 + \ldots + 853^2}{4} - \frac{3524^2}{16} = 777007 - 776161 = 846$$

The sum of squares for the blocks (batches) is given by

$$SS_B = \frac{750^2 + \ldots + 1006^2}{4} - \frac{3524^2}{16} = 786540 - 776161 = 10379$$

The residual sum of squares is given by

$$SS_E = SS_T - SS_M - SS_B = 13259 - 846 - 10379 = 2034$$

These calculations lead to the following ANOVA table.

Source of Variation	**Deg. of Freedom**	**Sum of Squares**	**Mean Square**	**Variance Ratio**
Extrusion Methods	3	846	282.00	1.25
Batches (blocks)	3	10379	3459.67	15.31
Residual (error)	9	2034	226.00	
Total	15	13259		

The following conclusions may be drawn.

Extrusion Methods

From F-tables $F_{3,9} = 3.86$ and since 1.25<3.86 we do not have sufficient evidence to reject the null hypothesis that the choice of extrusion method has no effect on the tensile strength of the tubes.

Batches (blocks)

From F-tables $F_{3,9} = 3.86$ and since 15.31>3.86 we have sufficient evidence to reject the null hypothesis that mean tensile strength is the same in all batches. We conclude that mean tensile strength does differ between batches.

Answers continued

3. The treatment totals are as follows:

Mechanism	A	B	C	D
Total	14.11	14.88	15.16	14.09

The row totals are as follows:

Angle	1	2	3	4
Total	15.82	14.91	13.97	13.54

The column totals are as follows:

Dummy	1	2	3	4
Total	14.53	13.65	15.98	14.08

The grand total is 58.24.
The total sum of squares is

$$3.84^2 + \cdots + 3.41^2 - \frac{58.24^2}{16} = 2.1568.$$

The treatments sum of squares is

$$\frac{1}{4}(14.11^2 + \cdots + 14.09^2) - \frac{58.24^2}{16} = 0.22145.$$

The angles sum of squares is

$$\frac{1}{4}(15.82^2 + \cdots + 13.54^2) - \frac{58.24^2}{16} = 0.77465.$$

The dummies sum of squares is

$$\frac{1}{4}(14.53^2 + \cdots + 14.08^2) - \frac{58.24^2}{16} = 0.76895.$$

The analysis of variance table is as follows.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Variance ratio
Treatments (mechanisms)	3	0.22145	0.07382	1.131
Angles	3	0.77465	0.25822	3.955
Dummies	3	0.76895	0.25632	3.926
Residual	6	0.39175	0.06529	
Total	15	2.15680		

The upper 5% point of the $F_{3,6}$ distribution is 4.76. Thus we can draw the following conclusions.

Answers continued

Treatments (mechanisms)

The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the mechanisms give equal mean maximum accelerations.

Angles

The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the angles have no effect on mean maximum accelerations.

(b) **Dummies**

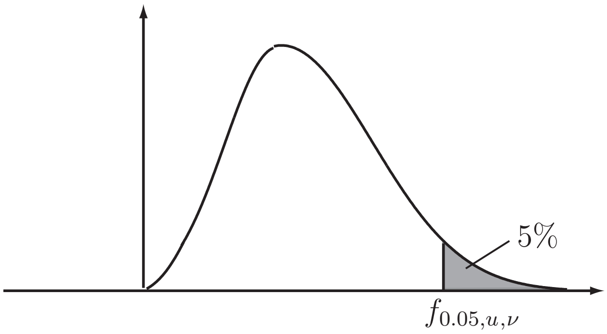
The variance ratio is not significant at the 5% level so we conclude that there is not sufficient evidence to reject the null hypothesis that the dummies have no effect on mean maximum accelerations.

The assumptions made are as follows.

- The observation in each cell is taken from a normal distributions.
- Each of these normal distributions has the same variance.
- The observations are independent.
- The effects of the factors are additive. That is, there are no interactions.

To allow investigation of the possibility of interactions we would need to make more than one observation in each cell.

Table 1: Percentage Points of the F Distribution (5% tail)



	Degrees of Freedom for the Numerator (u)														
ν	1	2	3	4	5	6	7	8	9	10	20	30	40	60	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	248.0	250.1	251.1	252.2	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.45	19.46	19.47	19.48	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.66	8.62	8.59	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.80	5.75	5.72	5.69	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.56	4.53	4.46	4.43	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.87	3.81	3.77	3.74	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.44	3.38	3.34	3.30	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.15	3.08	3.04	3.01	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	2.94	2.86	2.83	2.79	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.77	2.70	2.66	2.62	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.65	2.57	2.53	2.49	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.54	2.47	2.43	2.38	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.46	2.38	2.34	2.30	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.39	2.31	2.27	2.22	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.33	2.25	2.20	2.16	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.28	2.19	2.15	2.11	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.23	2.15	2.10	2.06	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.19	2.11	2.06	2.02	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.16	2.07	2.03	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.12	2.04	1.99	1.95	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.10	2.01	1.96	1.92	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.07	1.98	1.94	1.89	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.05	1.96	1.91	1.86	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.03	1.94	1.89	1.84	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.01	1.92	1.87	1.82	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	1.99	1.90	1.85	1.80	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	1.97	1.88	1.84	1.79	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	1.96	1.87	1.82	1.77	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	1.94	1.85	1.81	1.75	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	1.93	1.84	1.79	1.74	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.84	1.74	1.69	1.64	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.75	1.65	1.59	1.53	1.39
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.57	1.46	1.39	3.32	1.00