

Indices



Introduction

Indices, or powers, provide a convenient notation when we need to multiply a number by itself several times. In this Section we explain how indices are written, and state the rules which are used for manipulating them.

Expressions built up using non-negative whole number powers of a variable — known as polynomials — occur frequently in engineering mathematics. We introduce some common polynomials in this Section.

Finally, scientific notation is used to express very large or very small numbers concisely. This requires use of indices. We explain how to use scientific notation towards the end of the Section.



Prerequisites

Before starting this Section you should ...

- be familiar with algebraic notation and symbols



Learning Outcomes

On completion you should be able to ...

- perform calculations using indices
- state and use the laws of indices
- use scientific notation

1. Index notation

The number $4 \times 4 \times 4$ is written, for short, as 4^3 and read '4 raised to the power 3' or '4 cubed'. Note that the number of times '4' occurs in the product is written as a superscript. In this context we call the superscript 3 an **index** or **power**. Similarly we could write

$$5 \times 5 = 5^2, \text{ read '5 to the power 2' or '5 squared'}$$

and

$$7 \times 7 \times 7 \times 7 \times 7 = 7^5 \quad a \times a \times a = a^3, \quad m \times m \times m \times m = m^4$$

More generally, in the expression x^y , x is called the **base** and y is called the index or power. The plural of index is **indices**. The process of raising to a power is also known as **exponentiation** because yet another name for a power is an **exponent**. When dealing with numbers your calculator is able to evaluate expressions involving powers, probably using the x^y button.



Example 12

Use a calculator to evaluate 3^{12} .

Solution

Using the x^y button on the calculator check that you obtain $3^{12} = 531441$.



Example 13

Identify the index and base in the following expressions. (a) 8^{11} , (b) $(-2)^5$,
(c) p^{-q}

Solution

- (a) In the expression 8^{11} , 8 is the base and 11 is the index.
- (b) In the expression $(-2)^5$, -2 is the base and 5 is the index.
- (c) In the expression p^{-q} , p is the base and $-q$ is the index. The interpretation of a negative index will be given in sub-section 4 which starts on page 31.

Recall from Section 1.1 that when several operations are involved we can make use of the BODMAS rule for deciding the order in which operations must be carried out. The BODMAS rule makes no mention of exponentiation. Exponentiation should be carried out immediately after any brackets have been dealt with and before multiplication and division. Consider the following examples.



Example 14

Evaluate 7×3^2 .

Solution

There are two operations involved here, exponentiation and multiplication. The exponentiation should be carried out before the multiplication. So $7 \times 3^2 = 7 \times 9 = 63$.



Example 15

Write out fully (a) $3m^4$, (b) $(3m)^4$.

Solution

(a) In the expression $3m^4$ the exponentiation is carried out before the multiplication by 3. So

$$3m^4 \text{ means } 3 \times (m \times m \times m \times m) \text{ that is } 3 \times m \times m \times m \times m$$

(b) Here the bracketed expression is raised to the power 4 and so should be multiplied by itself four times:

$$(3m)^4 = (3m) \times (3m) \times (3m) \times (3m)$$

Because of the associativity of multiplication we can write this as

$$3 \times 3 \times 3 \times 3 \times m \times m \times m \times m \quad \text{or simply} \quad 81m^4.$$

Note the important distinction between $(3m)^4$ and $3m^4$.

Exercises

- Evaluate, without using a calculator, (a) 3^3 , (b) 3^5 , (c) 2^5 , (d) 0.2^2 , (e) 15^2 .
- Evaluate using a calculator (a) 7^3 , (b) $(14)^{3.2}$.
- Write each of the following using index notation:
 - $7 \times 7 \times 7 \times 7 \times 7$,
 - $t \times t \times t \times t$,
 - $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}$.
- Evaluate without using a calculator. Leave any fractions in fractional form.
 - $(\frac{2}{3})^2$,
 - $(\frac{2}{5})^3$,
 - $(\frac{1}{2})^2$,
 - $(\frac{1}{2})^3$,
 - 0.1^3 .

Answers

1. (a) 27, (b) 243, (c) 32, (d) 0.04, (e) 225
2. (a) 343, (b) 4651.7 (1 d.p.).
3. (a) 7^5 , (b) t^4 , (c) $\left(\frac{1}{2}\right)^2 \left(\frac{1}{7}\right)^3$
4. (a) $\frac{4}{9}$, (b) $\frac{8}{125}$, (c) $\frac{1}{4}$, (d) $\frac{1}{8}$, (e) 0.1^3 means $(0.1) \times (0.1) \times (0.1) = 0.001$

2. Laws of indices

There is a set of rules which enable us to manipulate expressions involving indices. These rules are known as the **laws of indices**, and they occur so commonly that it is worthwhile to memorise them.



Key Point 5

Laws of Indices

The laws of indices state:

First law: $a^m \times a^n = a^{m+n}$ add indices when multiplying numbers with the same base

Second law: $\frac{a^m}{a^n} = a^{m-n}$ subtract indices when dividing numbers with the same base

Third law: $(a^m)^n = a^{mn}$ multiply indices together when raising a number to a power

**Example 16**Simplify (a) $a^5 \times a^4$, (b) $2x^5(x^3)$.**Solution**

In each case we are required to multiply expressions involving indices. The bases are the same and we use the first law of indices.

(a) The indices must be added, thus $a^5 \times a^4 = a^{5+4} = a^9$.

(b) Because of the associativity of multiplication we can write

$$2x^5(x^3) = 2(x^5x^3) = 2x^{5+3} = 2x^8$$

The first law of indices (Key Point 5) extends in an obvious way when more terms are involved:

**Example 17**Simplify $b^5 \times b^4 \times b^7$.**Solution**

The indices are added. Thus $b^5 \times b^4 \times b^7 = b^{5+4+7} = b^{16}$.

**Task**Simplify $y^4y^2y^3$.**Your solution**

$$y^4y^2y^3 =$$

Answer

All quantities have the same base. To multiply the quantities together, the indices are added: y^9

**Example 18**

Simplify (a) $\frac{8^4}{8^2}$, (b) $x^{18} \div x^7$.

Solution

In each case we are required to divide expressions involving indices. The bases are the same and we use the second law of indices (Key Point 5).

(a) The indices must be subtracted, thus $\frac{8^4}{8^2} = 8^{4-2} = 8^2 = 64$.

(b) Again the indices are subtracted, and so $x^{18} \div x^7 = x^{18-7} = x^{11}$.



Simplify $\frac{5^9}{5^7}$.

Your solution

$$\frac{5^9}{5^7} =$$

Answer

The bases are the same, and the division is carried out by subtracting the indices: $5^{9-7} = 5^2 = 25$



Simplify $\frac{y^5}{y^2}$.

Your solution

$$\frac{y^5}{y^2} =$$

Answer

$$y^{5-2} = y^3$$

**Example 19**Simplify (a) $(8^2)^3$, (b) $(z^3)^4$.**Solution**

We use the third law of indices (Key Point 5).

(a) $(8^2)^3 = 8^{2 \times 3} = 8^6$

(b) $(z^3)^4 = z^{3 \times 4} = z^{12}$.

Simplify $(x^2)^5$.**Your solution**

$$(x^2)^5 =$$

Answer

$$x^{2 \times 5} = x^{10}$$

Simplify $(e^x)^y$ **Your solution**

$$(e^x)^y =$$

AnswerAgain, using the third law of indices, the two powers are multiplied: $e^{x \times y} = e^{xy}$

Two important results which can be derived from the laws of indices state:

**Key Point 6**Any non-zero number raised to the power 0 has the value 1, that is $a^0 = 1$ Any number raised to power 1 is itself, that is $a^1 = a$

A generalisation of the third law of indices states:



Key Point 7

$$(a^m b^n)^k = a^{mk} b^{nk}$$



Example 20

Remove the brackets from (a) $(3x)^2$, (b) $(x^3 y^7)^4$.

Solution

(a) Noting that $3 = 3^1$ and $x = x^1$ then $(3x)^2 = (3^1 x^1)^2 = 3^2 x^2 = 9x^2$

or, alternatively $(3x)^2 = (3x) \times (3x) = 9x^2$

(b) $(x^3 y^7)^4 = x^{3 \times 4} y^{7 \times 4} = x^{12} y^{28}$

Exercises

1. Show that $(-xy)^2$ is equivalent to $x^2 y^2$ whereas $(-xy)^3$ is equivalent to $-x^3 y^3$.
2. Write each of the following expressions with a single index:
(a) $6^7 6^9$, (b) $\frac{6^7}{6^{19}}$, (c) $(x^4)^3$
3. Remove the brackets from (a) $(8a)^2$, (b) $(7ab)^3$, (c) $7(ab)^3$, (d) $(6xy)^4$,
4. Simplify (a) $15x^2(x^3)$, (b) $3x^2(5x)$, (c) $18x^{-1}(3x^4)$.
5. Simplify (a) $5x(x^3)$, (b) $4x^2(x^3)$, (c) $3x^7(x^4)$, (d) $2x^8(x^{11})$, (e) $5x^2(3x^9)$

Answers

2. (a) 6^{16} , (b) 6^{-12} , (c) x^{12}
3. (a) $64a^2$, (b) $343a^3b^3$, (c) $7a^3b^3$, (d) $1296x^4y^4$
4. (a) $15x^5$, (b) $15x^3$, (c) $54x^3$
5. (a) $5x^4$, (b) $4x^5$, (c) $3x^{11}$, (d) $2x^{19}$, (e) $15x^{11}$

3. Polynomial expressions

An important group of mathematical expressions which use indices are known as **polynomials**. Examples of polynomials are

$$4x^3 + 2x^2 + 3x - 7, \quad x^2 + x, \quad 17 - 2t + 7t^4, \quad z - z^3$$

Notice that they are all constructed using non-negative whole number powers of the variable. Recall that $x^0 = 1$ and so the number -7 appearing in the first expression can be thought of as $-7x^0$. Similarly the 17 appearing in the third expression can be read as $17t^0$.



Key Point 8

Polynomials

A polynomial expression takes the form

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$$

where $a_0, a_1, a_2, a_3, \dots, a_n$ are all constants called the **coefficients** of the polynomial. The number a_0 is also called the **constant term**. The highest power in a polynomial is called the **degree** of the polynomial.

Polynomials with low degrees have special names and subscript notation is often not needed:

Polynomial	Degree	Name
$ax^3 + bx^2 + cx + d$	3	cubic
$ax^2 + bx + c$	2	quadratic
$ax + b$	1	linear
a	0	constant



Which of the following expressions are polynomials? Give the degree of those which are.

(a) $3x^2 + 4x + 2$, (b) $\frac{1}{x+1}$, (c) $\sqrt{x}$, (d) $2t + 4$,

(e) $3x^2 + \frac{4}{x} + 2$.

Recall that a polynomial expression must contain only terms involving non-negative whole number powers of the variable.

Give your answers by ringing the correct word (yes/no) and stating the degree if it is a polynomial.

Your solution

	polynomial		degree
(a) $3x^2 + 4x + 2$	yes	no	
(b) $\frac{1}{x+1}$	yes	no	
(c) $\sqrt{x}$	yes	no	
(d) $2t + 4$	yes	no	
(e) $3x^2 + \frac{4}{x} + 2$	yes	no	

Answer

(a) yes: polynomial of degree 2, called quadratic (b) no (c) no

(d) yes: polynomial of degree 1, called linear (e) no

Exercises

1. State which of the following are linear polynomials, which are quadratic polynomials, and which are constants.

(a) x , (b) $x^2 + x + 3$, (c) $x^2 - 1$, (d) $3 - x$, (e) $7x - 2$, (f) $\frac{1}{2}$,
(g) $\frac{1}{2}x + \frac{3}{4}$, (h) $3 - \frac{1}{2}x^2$.

2. State which of the following are polynomials.

(a) $-\alpha^2 - \alpha - 1$, (b) $x^{1/2} - 7x^2$, (c) $\frac{1}{x}$, (d) 19.

3. Which of the following are polynomials ?

(a) $4t + 17$, (b) $\frac{1}{2} - \frac{1}{2}t$, (c) 15, (d) $t^2 - 3t + 7$, (e) $\frac{1}{t^2} + \frac{1}{t} + 7$

4. State the degree of each of the following polynomials. For those of low degree, give their name.

(a) $2t^3 + 7t^2$, (b) $7t^7 + 14t^3 - 2t^2$, (c) $7x + 2$,
(d) $x^2 + 3x + 2$, (e) $2 - 3x - x^2$, (f) 42

Answers

- (a), (d), (e) and (g) are linear. (b), (c) and (h) are quadratic. (f) is a constant.
- (a) is a polynomial, (d) is a polynomial of degree 0. (b) and (c) are not polynomials.
- (a) (b) (c) and (d) are polynomials.
- (a) 3, cubic, (b) 7, (c) 1, linear, (d) 2, quadratic, (e) 2, quadratic, (f) 0, constant.

4. Negative indices

Sometimes a number is raised to a negative power. This is interpreted as follows:



Key Point 9

Negative Powers

$$a^{-m} = \frac{1}{a^m}, \quad a^m = \frac{1}{a^{-m}}$$

Thus a negative index can be used to indicate a reciprocal.



Example 21

Write each of the following expressions using a positive index and simplify if possible.

(a) 2^{-3} , (b) $\frac{1}{4^{-3}}$, (c) x^{-1} , (d) x^{-2} , (e) 10^{-1}

Solution

(a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, (b) $\frac{1}{4^{-3}} = 4^3 = 64$, (c) $x^{-1} = \frac{1}{x^1} = \frac{1}{x}$, (d) $x^{-2} = \frac{1}{x^2}$,
 (e) $10^{-1} = \frac{1}{10^1} = \frac{1}{10}$ or 0.1.



Write each of the following using a positive index. Use Key Point 9.

(a) $\frac{1}{t^{-4}}$, (b) 17^{-3} , (c) y^{-1} , (d) 10^{-2}

Your solution

(a) $\frac{1}{t^{-4}} =$

Answer

t^4

Your solution

(b) $17^{-3} =$

Answer

$$\frac{1}{17^3}$$

Your solution

(c) $y^{-1} =$

Answer

$$\frac{1}{y}$$

Your solution

(d) $10^{-2} =$

Answer

$\frac{1}{10^2}$ which equals $\frac{1}{100}$ or 0.01



Simplify $\frac{a^8 \times a^7}{a^4}$

Use the first law of indices to simplify the numerator:

Your solution

$$\frac{a^8 \times a^7}{a^4} =$$

Answer

$$\frac{a^{15}}{a^4}$$

Now use the second law to simplify the result:

Your solution

Answer

$$a^{11}$$



Simplify $\frac{m^9 \times m^{-2}}{m^{-3}}$

First simplify the numerator using the first law of indices:

Your solution

$$\frac{m^9 \times m^{-2}}{m^{-3}} =$$

Answer

$$\frac{m^7}{m^{-3}}$$

Then use the second law to simplify the result:

Your solution

Answer

$$m^{7-(-3)} = m^{10}$$

Exercises

1. Write the following numbers using a positive index and also express your answers as decimal fractions:

(a) 10^{-1} , (b) 10^{-3} , (c) 10^{-4}

2. Simplify as much as possible:

(a) x^3x^{-2} , (b) $\frac{t^4}{t^{-3}}$, (c) $\frac{y^{-2}}{y^{-6}}$.

Answers

1. (a) $\frac{1}{10} = 0.1$, (b) $\frac{1}{10^3} = 0.001$, (c) $\frac{1}{10^4} = 0.0001$.
2. (a) $x^1 = x$, (b) $t^{4+3} = t^7$, (c) $y^{-2+6} = y^4$.

5. Fractional indices

So far we have used indices that are whole numbers. We now consider fractional powers. Consider the expression $(16^{\frac{1}{2}})^2$. Using the third law of indices, $(a^m)^n = a^{mn}$, we can write

$$(16^{\frac{1}{2}})^2 = 16^{\frac{1}{2} \times 2} = 16^1 = 16$$

So $16^{\frac{1}{2}}$ is a number which when squared equals 16, that is 4 or -4 . In other words $16^{\frac{1}{2}}$ is a square root of 16. There are always two square roots of a non-zero positive number, and we write $16^{\frac{1}{2}} = \pm 4$



Key Point 10

In general $a^{\frac{1}{2}}$ is a square root of a $a \geq 0$

Similarly

$$(8^{\frac{1}{3}})^3 = 8^{\frac{1}{3} \times 3} = 8^1 = 8$$

so that $8^{\frac{1}{3}}$ is a number which when cubed equals 8. Thus $8^{\frac{1}{3}}$ is the cube root of 8, that is $\sqrt[3]{8}$, namely 2. Each number has only one cube root, and so

$$8^{\frac{1}{3}} = 2$$

In general



Key Point 11

$a^{\frac{1}{3}}$ is the cube root of a

More generally we have



Key Point 12

The n th root of a is denoted by $a^{\frac{1}{n}}$.

When $a < 0$ the n th root only exists if n is odd.

If $a > 0$ the positive n th root is denoted by $\sqrt[n]{a}$

If $a < 0$ the negative n th root is $-\sqrt[n]{|a|}$

Your calculator will be able to evaluate fractional powers, and roots of numbers. Check that you can obtain the results of the following Examples on your calculator, but be aware that calculators normally give only one root when there may be others.



Example 22

Evaluate (a) $144^{1/2}$, (b) $125^{1/3}$

Solution

- (a) $144^{1/2}$ is a square root of 144, that is ± 12 .
 (b) Noting that $5^3 = 125$, we see that $125^{1/3} = \sqrt[3]{125} = 5$



Example 23

Evaluate (a) $32^{1/5}$, (b) $32^{2/5}$, (c) $8^{2/3}$.

Solution

- (a) $32^{1/5}$ is the 5th root of 32, that is $\sqrt[5]{32}$. Now $2^5 = 32$ and so $\sqrt[5]{32} = 2$.
 (b) Using the third law of indices we can write $32^{2/5} = 32^{2 \times \frac{1}{5}} = (32^{1/5})^2$. Thus

$$32^{2/5} = ((32)^{1/5})^2 = 2^2 = 4$$

- (c) Note that $8^{1/3} = 2$. Then

$$8^{2/3} = 8^{2 \times \frac{1}{3}} = (8^{1/3})^2 = 2^2 = 4$$

Note the following alternatives:

$$8^{2/3} = (8^{1/3})^2 = (8^2)^{1/3}$$



Example 24

Write the following as a simple power with a single index:

- (a) $\sqrt{x^5}$, (b) $\sqrt[4]{x^3}$.

Solution

- (a) $\sqrt{x^5} = (x^5)^{1/2}$. Then using the third law of indices we can write this as $x^{5 \times \frac{1}{2}} = x^{5/2}$.
 (b) $\sqrt[4]{x^3} = (x^3)^{1/4}$. Using the third law we can write this as $x^{3 \times \frac{1}{4}} = x^{3/4}$.



Example 25

Show that $z^{-1/2} = \frac{1}{\sqrt{z}}$.

Solution

$$z^{-1/2} = \frac{1}{z^{1/2}} = \frac{1}{\sqrt{z}}$$



Simplify $\frac{\sqrt{z}}{z^3 z^{-1/2}}$

First, rewrite $\sqrt{z}$ using an index and simplify the denominator using the first law of indices:

Your solution

$$\frac{\sqrt{z}}{z^3 z^{-1/2}} =$$

Answer

$$\frac{z^{\frac{1}{2}}}{z^{\frac{5}{2}}}$$

Finally, use the second law to simplify the result:

Your solution

Answer

$$z^{\frac{1}{2} - \frac{5}{2}} = z^{-2} \text{ or } \frac{1}{z^2}$$



Example 26

The generalisation of the third law of indices states that $(a^m b^n)^k = a^{mk} b^{nk}$. By taking $m = 1$, $n = 1$ and $k = \frac{1}{2}$ show that $\sqrt{ab} = \sqrt{a} \sqrt{b}$.

Solution

Taking $m = 1$, $n = 1$ and $k = \frac{1}{2}$ gives $(ab)^{1/2} = a^{1/2} b^{1/2}$.

Taking the case when all these roots are positive, we have $\sqrt{ab} = \sqrt{a} \sqrt{b}$.



Key Point 13

$$\sqrt{ab} = \sqrt{a} \sqrt{b} \quad a \geq 0, \quad b \geq 0$$

This result often allows answers to be written in alternative forms. For example, we may write $\sqrt{48}$ as $\sqrt{3 \times 16} = \sqrt{3} \sqrt{16} = 4\sqrt{3}$.

Although this rule works for multiplication we should be aware that it does **not** work for addition or subtraction so that

$$\sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b}$$

Exercises

- Evaluate using a calculator (a) $3^{1/2}$, (b) $15^{-\frac{1}{3}}$, (c) 85^3 , (d) $81^{1/4}$
- Evaluate using a calculator (a) 15^{-5} , (b) $15^{-2/7}$
- Simplify (a) $\frac{a^{11} a^{3/4}}{a^{-1/2}}$, (b) $\frac{\sqrt{z}}{z^{3/2}}$, (c) $\frac{z^{-5/2}}{\sqrt{z}}$, (d) $\frac{\sqrt[3]{a}}{\sqrt[2]{a}}$, (e) $\frac{\sqrt[5]{z}}{z^{1/2}}$.
- Write each of the following expressions with a single index:

$$(a) (x^{-4})^3, \quad (b) x^{1/2} x^{1/4}, \quad (c) \frac{x^{1/2}}{x^{1/4}}$$

Answers

- (a) 1.7321, (b) 0.4055, (c) 614125, (d) 3
- (a) 0.000001317 (4 s.f.), (b) 0.4613 (4 s.f.),
- (a) $a^{12.25}$, (b) z^{-1} , (c) z^{-3} , (d) $a^{-1/6}$, (e) $z^{-3/10}$
- (a) x^{-12} , (b) $x^{3/4}$, (c) $x^{1/4}$

6. Scientific notation

It is often necessary to use very large or very small numbers such as 78000000 and 0.00000034. **Scientific notation** can be used to express such numbers in a more concise form. Each number is written in the form

$$a \times 10^n$$

where a is a number between 1 and 10. We can make use of the following facts:

$$10 = 10^1, \quad 100 = 10^2, \quad 1000 = 10^3 \quad \text{and so on}$$

and

$$0.1 = 10^{-1}, \quad 0.01 = 10^{-2}, \quad 0.001 = 10^{-3} \quad \text{and so on.}$$

For example,

- the number 5000 can be written $5 \times 1000 = 5 \times 10^3$
- the number 403 can be written $4.03 \times 100 = 4.03 \times 10^2$
- the number 0.009 can be written $9 \times 0.001 = 9 \times 10^{-3}$

Furthermore, to multiply a number by 10^n the decimal point is moved n places to the right if n is a positive integer, and n places to the left if n is a negative integer. (If necessary additional zeros are inserted to make up the required number of digits before the decimal point.)



Write the numbers 0.00678 and 123456.7 in scientific notation.

Your solution

Answer

$$0.00678 = 6.78 \times 10^{-3} \quad 123456.7 = 1.234567 \times 10^5$$

Engineering constants

Many constants appearing in engineering calculations are expressed in scientific notation. For example the charge on an electron equals 1.6×10^{-19} coulomb and the speed of light is $3 \times 10^8 \text{ m s}^{-1}$. Avogadro's constant is equal to 6.023×10^{26} and is the number of atoms in one kilomole of an element. Clearly the use of scientific notation avoids writing lengthy strings of zeros.

Your scientific calculator will be able to accept numbers in scientific notation. Often the E button is used and a number like 4.2×10^7 will be entered as $4.2E7$. Note that $10E4$ means 10×10^4 , that is 10^5 . To enter the number 10^3 say, you would key in $1E3$. Entering powers of 10 incorrectly is a common cause of error. You must check how your particular calculator accepts numbers in scientific notation.

The following Task is designed to check that you can enter numbers given in scientific notation into your calculator.



Use your calculator to find $4.2 \times 10^{-3} \times 3.6 \times 10^{-4}$.

Your solution

$$4.2 \times 10^{-3} \times 3.6 \times 10^{-4} =$$

Answer

$$1.512 \times 10^{-6}$$

Exercises

1. Express each of the following numbers in scientific notation:

- (a) 45, (b) 456, (c) 2079, (d) 7000000, (e) 0.1, (f) 0.034,
(g) 0.09856

2. Simplify $6 \times 10^{24} \times 1.3 \times 10^{-16}$

Answers

1. (a) 4.5×10^1 , (b) 4.56×10^2 , (c) 2.079×10^3 , (d) 7×10^6 , (e) 1×10^{-1} ,
(f) 3.4×10^{-2} , (g) 9.856×10^{-2}
2. 7.8×10^8