Chocolate – a sweet application of structural concepts

Shahad Shah

One example in everyday life in which a structural concept is used is chocolate. This might sound crazy (and greedy) but figure 1-3 should help you realise how they do if you haven't realised already that is.

Figure 1

Figure 2

Figure 3

Many chocolate bars cleverly use structural concepts such as stress distribution, different cross sections, shear and bending for ease of production and a memorable feature to their product.

To snap a finger of a Kit Kat bar, the consumer applies a bending moment which moves one finger in a direction opposite to the direction of the finger on the other side of the ridge. Furthermore during the short time that the force is applied there will be some bending action on the ridge. This will result in elongation on the top of the ridge and shortening on the bottom. The normal stresses on any cross section are distributed linearly with the maximum normal stresses occurring on the surface furthest from the neutral plane. This plays a part in the stress distribution. The formula for normal bending stress is shown below where M is the bending moment, y is the distance from the neutral axis and I is the second moment of area.

$$\sigma_{bending} = -\frac{My}{I}$$

Source: http://www.ecourses.ou.edu/cgibin/ebook.cgi?doc=&topic=me&chap_sec=04.1&page=theory

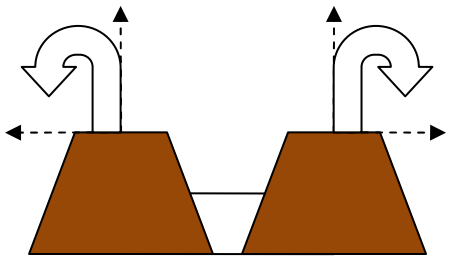
Finally, stress is concentrated in the ridge because it has the smallest cross sectional area in the bar and therefore stress becomes concentrated here. For a given force applied the smaller the area, the higher the stress. The fundamental formula is shown below where F is force and A is the area.

$$\text{Stress} = \sigma = \frac{F}{A}$$

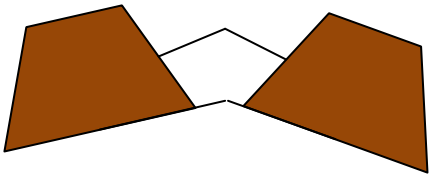
Source: http://www.engineersedge.com/material_science/stress_definition.htm

This process is shown in the diagram below.

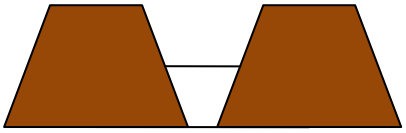
Bending Moment
Fingers-brown
Ridge-white
Moment-solid arrows
Force component – dashed arrow



Bending
Shows exaggerated elongation at top and shortening at bottom of ridge before eventual brittle failure



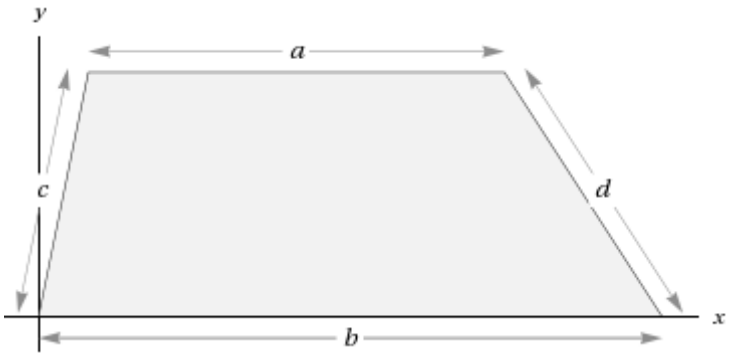
Stress Distribution
Stress concentrated at ridge (white area) as smaller cross section therefore larger stress.



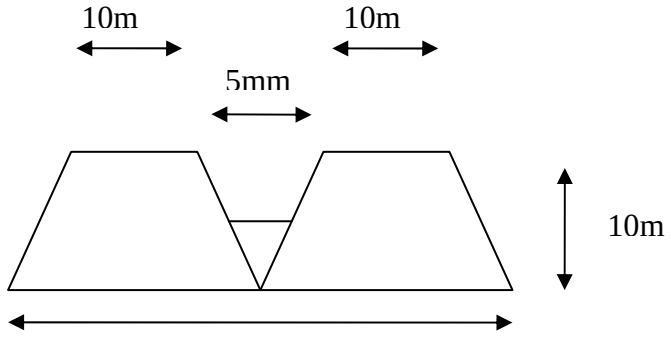
Calculating the moment of inertia of a Kit Kat and the force needed to break it
This will be found mathematically. The formula of the second moment of area for a trapezoid is:

$$I_{xx} = -\frac{(3a+b)\,((a-b+c-d)\,(a-b-c+d)\,(a-b+c+d)\,(-a+b+c+d))^{3/2}}{96\,(a-b)^3}$$

The corresponding trapezoid and axes being:



Measuring the dimensions of a typical Kit Kat gives the diagram below:



Length=94mm

30mm

The shape is above undergraduate level so I shall work out the moment of inertia for one finger. Using the above formula and approximate dimensions a,c,d=10mm and b=15mm give the second moment of area to be $I_{xx}=3403mm^4$.
Source: http://www.wolframalpha.com/input/?i=second+moment+of+area+of+trapezoid

Appendices

Images from:
Figure 1- http://www.pimpthatsnack.com/snackabase/kitkat/17
Figure 2- http://www.coffeebuyer.co.uk/mall/productpage.cfm/thecoffeebuyer/_35776/344373/Toblerone%20Giant%20Milk%20Chocolate%20Bar%20(1%20x%204.5kg)
Figure 3- http://www.chocablog.com/reviews/cadbury-dairy-milk-with-cranberry-granola/

References

www.structuralconcepts.org (Last accessed 7/2/11)
Mechanics-Theory
http://www.ecourses.ou.edu/cgibin/ebook.cgi?doc=&topic=me&chap_sec=04.1&page=theory (Last accessed 7/2/11)
Mechanics of materials
http://www.engineersedge.com/material_science/stress_definition.htm (Last accessed 7/2/11)
Wolfram Alpha
http://www.wolframalpha.com/input/?i=second+moment+of+area+of+trapezoid (Last accessed 7/2/11)