Torsion and Shear Stress in an Oreo

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Consider an Oreo biscuit with the cream in the middle, subjecting the two outer biscuits to a twisting moment may result in shear stress developing in the cream. The thickness of the cream is very small compared to the diameter of the biscuit; therefore the maximum shear stress occurs in the throat area.

Thus for a given torque applied to the biscuit the maximum shear stress in the cream is:

$$\tau_{\max} = \frac{T(\frac{d}{2} + t_{throat})}{I_p}$$

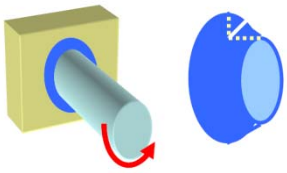


Figure 1[2]

Where T = torque applied
d = outer diameter of the biscuit
t_{throat} = throat thickness
I_p = polar moment of area of throat thickness

$$I_p = \frac{\pi}{32}[(d + 2t_{throat})^4 - d^4]$$

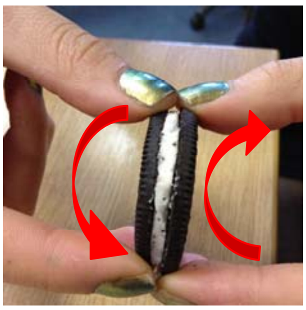


Figure A

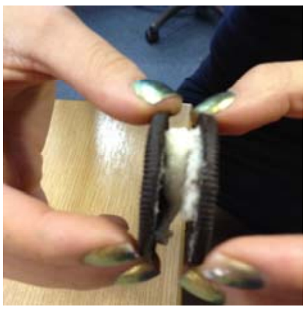


Figure B

Figure C

The cream section, in figure B demonstrates how as the section is subjected to couples it will deform in such a way that planes perpendicular to its axis before loadings remain plane and perpendicular to the axis after loading. In addition, radial lines in the cross section remain radial and the length does not change appreciably.

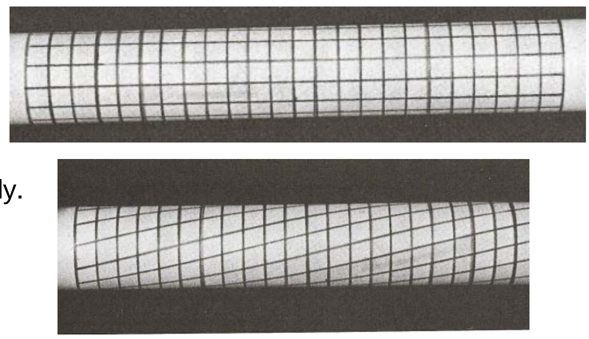
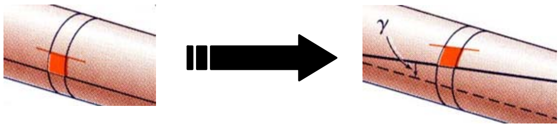
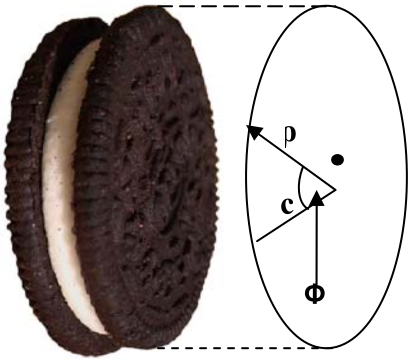


Figure 2[3]

As a torsional load is applied to the circular outer biscuit, the interior cream deforms into a rhombus cross section.



Since the ends of the element, or the biscuits, remain planar the shear strain is equal to the angle of twist.



It follows that:

$$L\gamma = \rho\phi \quad \text{or} \quad \gamma = \frac{\rho\phi}{L}$$

Shear Strain is proportional to twist and radius:

$$\gamma_{max} = \frac{c\phi}{L} \quad \text{and} \quad \gamma = \frac{\rho}{c}\gamma_{max}$$

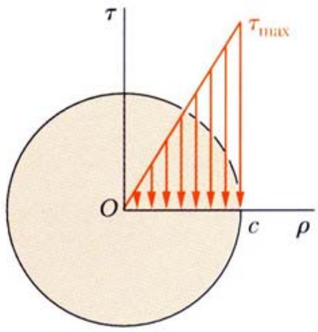
Multiplying by shear modulus:

$$G\gamma = \frac{\rho}{c}G\gamma_{max}$$

Using Hooke's Law

$$\tau = G\gamma$$

Therefore → $$\tau = \frac{\rho}{c}\tau_{max}$$



The shearing stress varies linearly with the radial position within the section.
The torsion is given by the elastic torsion formula;

$$\tau_{max} = \frac{Tc}{J} \quad \text{and} \quad \tau = \frac{T\rho}{J}$$

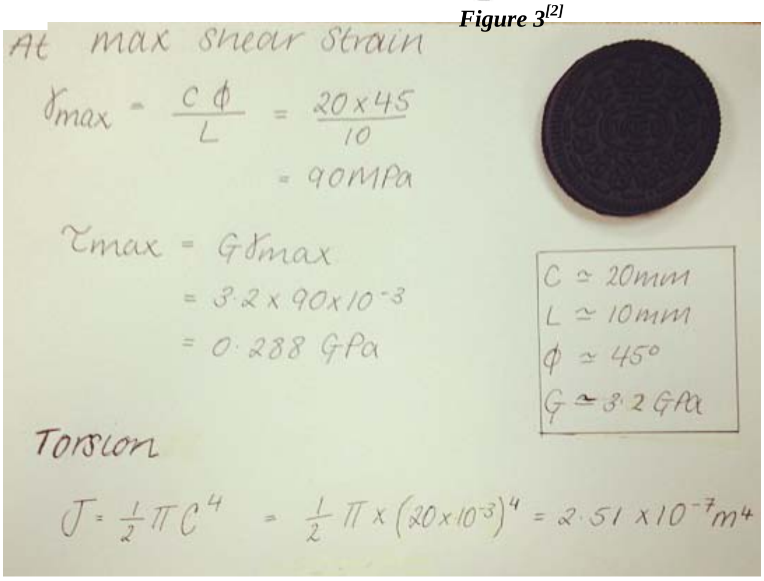
$$J = \frac{1}{2}\pi c^4$$

Figure 3[2]

In an Oreo

When subjected to torsion a brittle specimen breaks along planes perpendicular to the direction in which tension is a maximum ie: 45 degrees to the axis.

[for chalk G = 3.2 GPa]



References

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