What is the sweet spot on a bat and how is it found?

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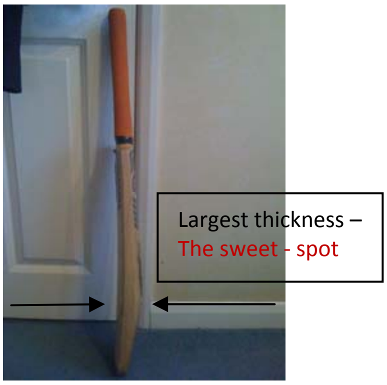
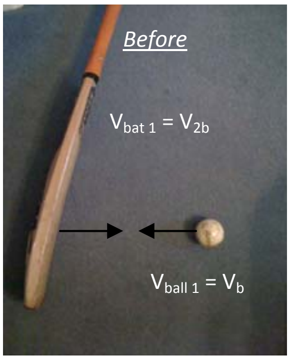
Description of ball–bat collision

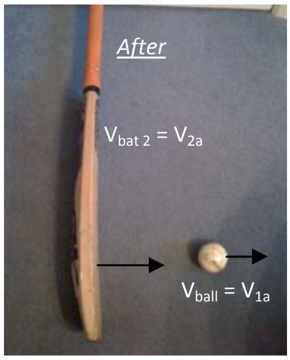
- Large forces in small time – Over 2 Tonnes in less than 1ms
- The ball compresses against the bat, stops and then expands
- The bat bends and compresses
- The conservation of energy is from kinetic energy > potential energy > kinetic energy
- Some energy dissipated to sound, heat, distortion of the ball and vibrations in the bat

Batsmen generally try to hit the ball at the 'middle' of the bat. This is not the literal meaning, it is situated near the end of the bat where the largest thickness of the bat is. It is also known as the sweet spot.



Bat – Ball Collision Diagrams





Centre of percussion & Newton's second law

When the ball hits the bat, it causes the bat to be pushed backwards but due to the forward rotation of the batsman's hands (to strike the ball high) a forward force is also present and therefore two opposite forces of different values cause a 'sting' to the batsman's hands.

This making it harder to play far shots as it would hurt too much!

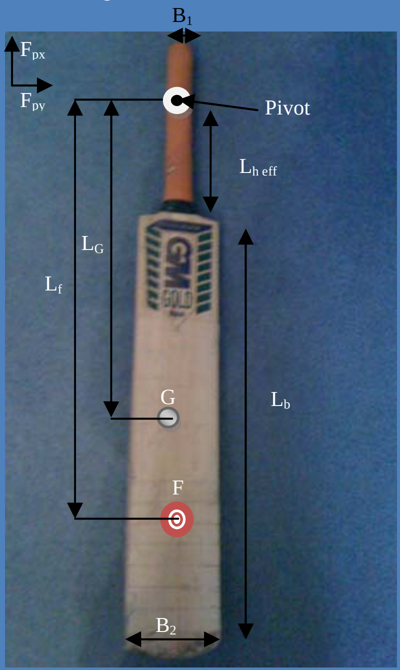
If however the ball is struck at 'the middle of the bat' which is scientifically known as the centre of percussion, a vibration occurs at the fundamental node of the bat, both forces mentioned before cancel out and there is no force felt in the batsman's hands.

He can now hit the ball as fast as he wants.

A more detailed analysis of this including a complex vibration analysis is used on computers to help produce and innovate modern day cricket bats. This is similarly done for tennis rackets and baseball bats.

Calculation of sweet spot distance from handle – (on my bat!)

Assuming bat is slender and has a pivoted end:



F_{px} is the x component of the force exerted on the bat by the batsman =0

F_{py} is the y component of the force exerted on the bat by the batsman =0

L_b + L_{heff} is the total length of the bat = 500mm + 280mm respectively

L_G is the length from the pivot to the centre of mass

L_f is the length from the pivot to the contact point 'F' where the ball meets the bat

Mass of the bat m = 1.2kg

B_1 and B_2 are the widths of the handle (assuming it is slender) and the bat respectively = 30mm & 110mm

The length L_f determined below…

Sum of the moments at the pivot = $I_p\alpha = F \times L_f$ (where α is the angular acceleration = a_{GX}/l_G)

$$L_G = \frac{140.(280).(30) + 530.(500).(110)}{(280).(30) + (500).(110)} = 481\text{mm}$$

$$I_p = (1/12)mL^2 + m(L_G)^2 = (1/12).(1.2 \text{x} 880^2) + 1.2.(481)^2 = 355073.2\text{mm}^4$$

moment of inertia parallel axis theorem Newton's second law

If (1) $\Sigma M_p = I_p\alpha = FL_f$ and (2) F_{px} (=0) + F = ma_{Gx} substituting in $\alpha = a_{GX}/l_G$ in (2) we get $F = m\alpha l_G$

Substituting this into (1) via α we get $(FL_f/I_p)mL_G$= F ,which equals $L_f = I_p/mL_G$

Substituting the known values in we get L_f = (355073.2/1.2 x 481) = 615.2 mm

Q: Im not so good with numbers – how can I determine it without using any!

Centre of percussion – *determined practically*

The centre of percussion can also be estimated practically without using any calculations. To find the corresponding node to the fundamental mode, grip with your fingers and thumb about six inches from the top of the handle. Hit the bat at various points with an impact hammer. The point where you feel no vibration and hear almost nothing is the node or the centre of percussion. If the bat is hit below the centre of mass but above/below the centre of percussion, the hand will feel a jolt. If it is hit at the centre of percussion, the hand will feel nothing.

Understanding and Using Structural Concepts

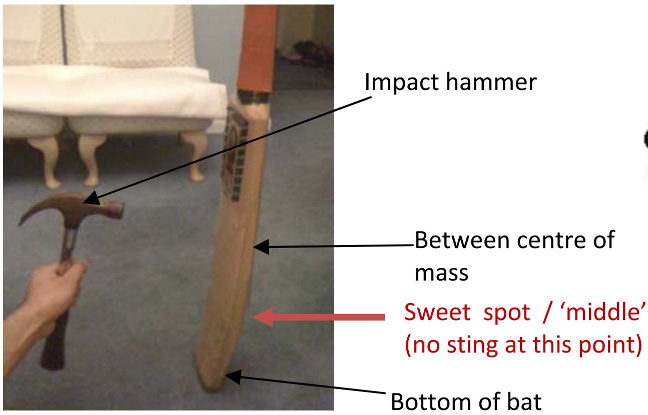
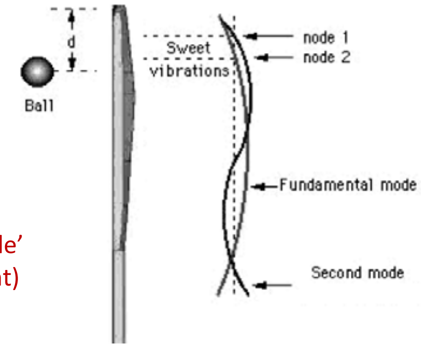


Illustration of COP experiment



Fundamental node Diagram – [1]

Q: Which is more important when trying to hit a ball far? - The velocity my bat hits the ball or the velocity of the incoming ball?

Conservation of momentum & energy

In the simplest form of - we will take the problem to be a one-dimensional problem.
The mass of the bat and ball, and their respective velocities can be related via linear conservation of momentum. (m_1 = mass of the ball and m_2 = mass of the bat)

Equating before and after collision: $m_1v_{1b} + m_2v_{2b} = m_1v_{1a} + m_2v_{2a}$ (1)

The conservation of energy theorem relates kinetic energy, potential energy, and non-conservative forces (like friction) which act on the system. During collision, the ball undergoes compression, and damping forces dissipate much of the ball's initial kinetic energy into the forms listed at the beginning. These energies are related between the elastic properties of the ball and the relative velocities of the bat and ball – known as the ***coefficient of restitution (e).***

$$e = \frac{\text{relative speed after collision}}{\text{relative speed before collision}} = \frac{v_{1a} - v_{2a}}{v_{1b} - v_{2b}} \quad (2)$$

Modern day cricket balls are designed to have an e value of around 0.58

Rearranging equation 1 to make v_{2a} the subject as we are not interested in the bat speed after collision: We get:

$$V_{2a} = (e(v_{1b} - v_{2b}) + v_{1a}) = m_1v_{1b} + m_2v_{2b} = m_1v_{1a} + m_2v_{2a} = m_1v_{1b} - m_1v_{1a} = m_2v_{2a} - m_2v_{2b}$$

substituting the above in equation 2 (changing signs and re-arranging)

$$m_1v_{1b} - m_1v_{1a} = -m_2v_{2b} + m_2ev_{1b} - m_2ev_2b + m_2v_{1a}$$
$$m_1v_{1a} + m_2v_{1a} = m_1v_{1b} - m_2ev_1b + m_2v_{2b} + m_2ev_{2b}$$

$$v_{1a} = \frac{v_{1b}(m_1 - m_2e) + v_{2b}(m_2 + m_2e)}{m_1 + m_2}$$ = exit speed of ball after collision

Here we can replace $(m_1-m_2e)/(m_1+m_2)$ with q (material factor) so now the exit speed equation becomes:
$$\mathbf{v_{1a} = qv_{1b} + (1+q)v_{2b}}$$

This equation shows that the velocity of the bat striking the ball is more significant than the speed of the approaching ball.

References

Image:[1] - http://www.vibrationdata.com/cricket_bat.pdf
Websites
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http://www.physics.usyd.edu.au/~cross/cricket.html
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